

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



Level 2 Certificate in Further Mathematics  
June 2012

## Further Mathematics

8360/2

## Level 2

## Paper 2 Calculator

Friday 1 June 2012 1.30 pm to 3.30 pm

For this paper you must have:

- a calculator
- mathematical instruments.



### Time allowed

- 2 hours

### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
TOTAL	

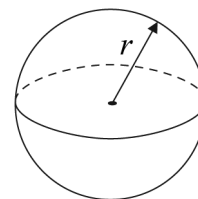


J U N 1 2 8 3 6 0 2 0 1

## Formulae Sheet

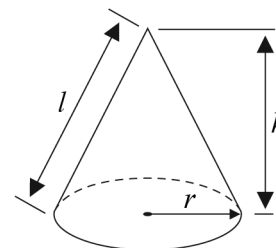
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area of cone} = \pi r l$$



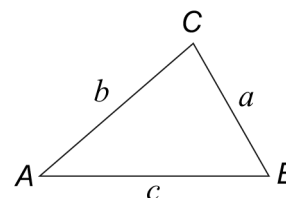
In any triangle  $ABC$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

$$\text{Sine rule} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

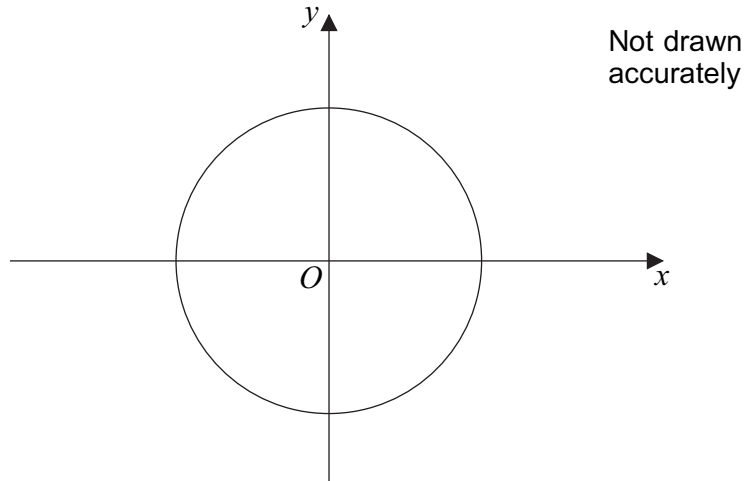
### Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$



Answer **all** questions in the spaces provided.

**1** Here is a sketch of the circle  $x^2 + y^2 = 36$



Work out the circumference of the circle.

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Answer..... (3 marks)

**Turn over for the next question**



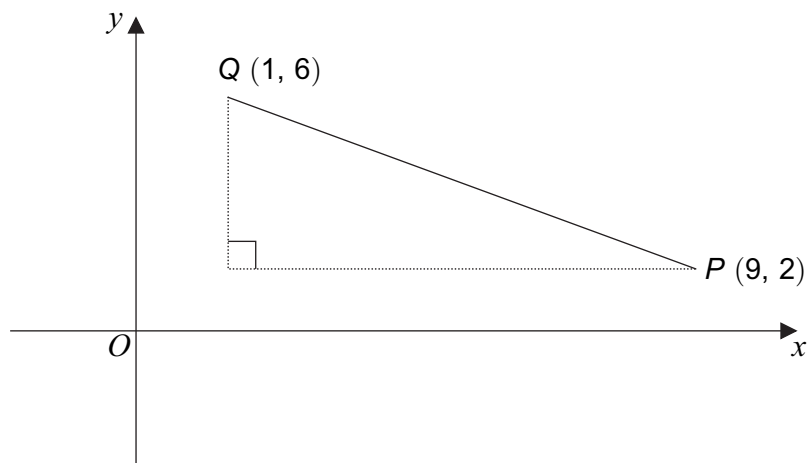
2

$$y = 5x^3 - 4x^2$$

Work out  $\frac{dy}{dx}$ .

$$\frac{dy}{dx} = \dots\dots\dots (2 \text{ marks})$$

3



Not drawn  
accurately

Work out the length of  $PQ$ .  
Give your answer to 3 significant figures.

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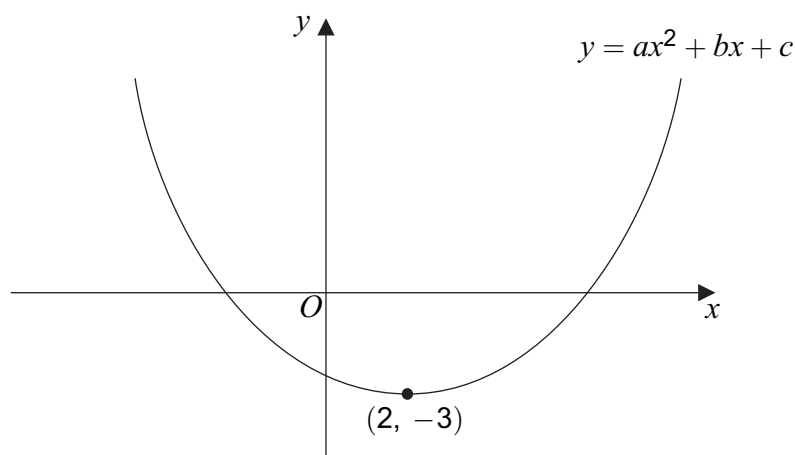
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$$PQ = \dots\dots\dots (4 \text{ marks})$$



- 4 A sketch of  $y = ax^2 + bx + c$  is shown.  
The minimum point is  $(2, -3)$ .

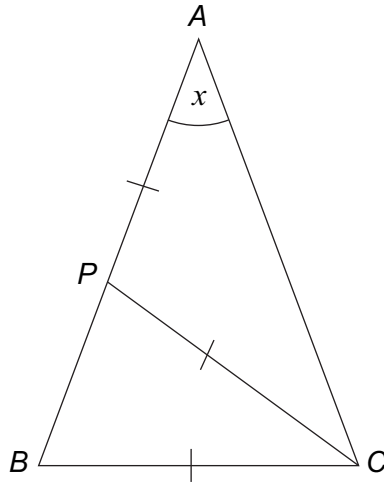


For the sketch shown, circle the correct answer in each of the following.

- 4 (a) The value of  $a$  is  
zero                      positive                      negative                      (1 mark)
- 4 (b) The value of  $c$  is  
zero                      positive                      negative                      (1 mark)
- 4 (c) The solutions of  $ax^2 + bx + c = 0$  are  
both zero                      both positive                      both negative                      one positive and  
one negative                      (1 mark)
- 4 (d) The **number** of solutions of  $ax^2 + bx + c = -6$  is  
0                      1                      2                      3                      (1 mark)
- 4 (e) The equation of the tangent to  $y = ax^2 + bx + c$  at  $(2, -3)$  is  
 $x = 2$                        $y = 2$                        $x = -3$                        $y = -3$                       (1 mark)



- 5**  $ABC$  is a triangle.  
 $P$  is a point on  $AB$  such that  $AP = PC = BC$   
 Angle  $BAC = x$



Not drawn accurately

- 5 (a)** Prove that angle  $ABC = 2x$

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(3 marks)

- 5 (b)** You are also given that  $AB = AC$

Work out the value of  $x$ .

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 .....  
 .....

$x =$  ..... degrees (3 marks)



6 (a) Expand  $3x(2x - 5y)$

Answer..... (2 marks)

6 (b) Expand and simplify  $(3x + 2y)(3x - 4y)$

.....  
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Answer..... (3 marks)

6 (c) Work out the ratio  $(3x + 2y)(3x - 4y) : 3x(2x - 5y)$  when  $y = 0$

Give your answer as simply as possible.

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Answer..... : ..... (2 marks)

7  $1 \leq m \leq 5$  and  $-9 \leq n \leq 2$

7 (a) Work out an inequality for  $m + n$ .

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Answer.....  $\leq m + n \leq$  ..... (2 marks)

7 (b) Work out an inequality for  $(m + n)^2$ .

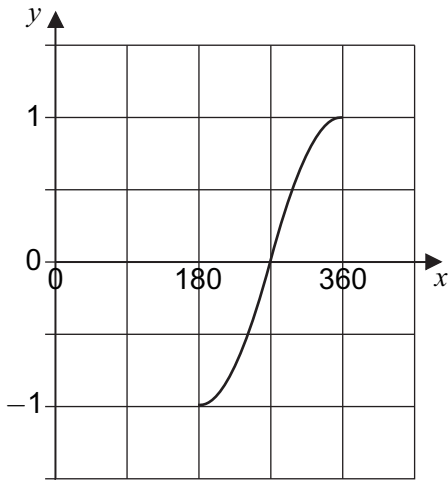
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Answer.....  $\leq (m + n)^2 \leq$  ..... (2 marks)

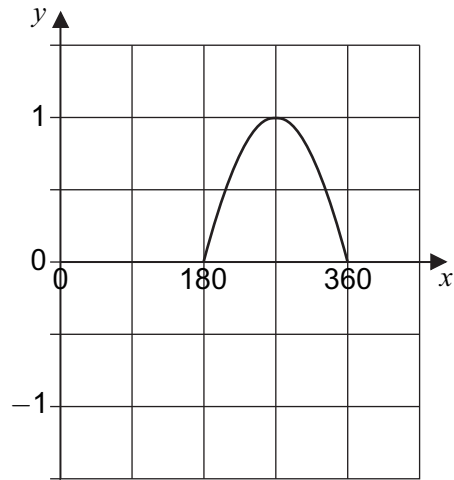


8 Four graphs are shown for  $180^\circ \leq x \leq 360^\circ$

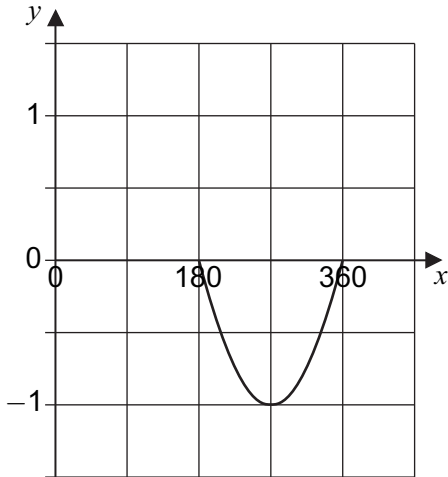
**Graph A**



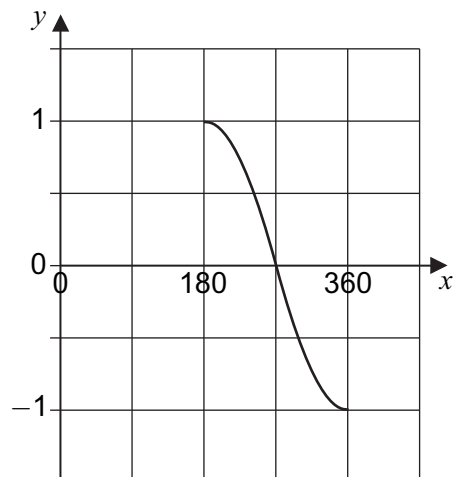
**Graph B**



**Graph C**



**Graph D**



8 (a) Which graph is  $y = \sin x$ ?

Graph ..... (1 mark)

8 (b) Which graph is  $y = \cos x$ ?

Graph ..... (1 mark)





9 Here is a formula.

$$5t + 3 = 4w(t + 2)$$

9 (a) Rearrange the formula to make  $t$  the subject.

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Answer..... (4 marks)

9 (b) Work out the exact value of  $t$  when  $w = -\frac{1}{8}$

Give your answer in its simplest form.

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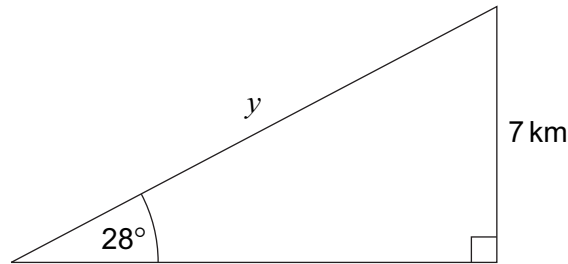
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$t =$  ..... (3 marks)



10

An aircraft flies  $y$  kilometres in a straight line at an angle of elevation of  $28^\circ$ .  
The gain in height is 7 kilometres.



Not drawn  
accurately

Work out the value of  $y$ .

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$y = \dots\dots\dots$  km (3 marks)

11

A sphere has radius  $x$  centimetres.  
A hemisphere has radius  $y$  centimetres.  
The shapes have equal volumes.

Work out the value of  $\frac{y}{x}$ .

Give your answer in the form  $a^{\frac{1}{3}}$  where  $a$  is an integer.

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$\frac{y}{x} = \dots\dots\dots$  (3 marks)



12

Expand and simplify  $(t + 4)^3$

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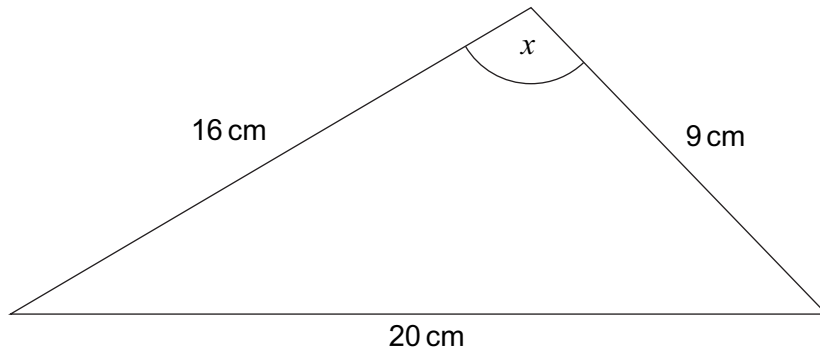
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Answer..... (3 marks)

13



Not drawn accurately

Work out angle  $x$ .

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$x =$  ..... degrees (3 marks)

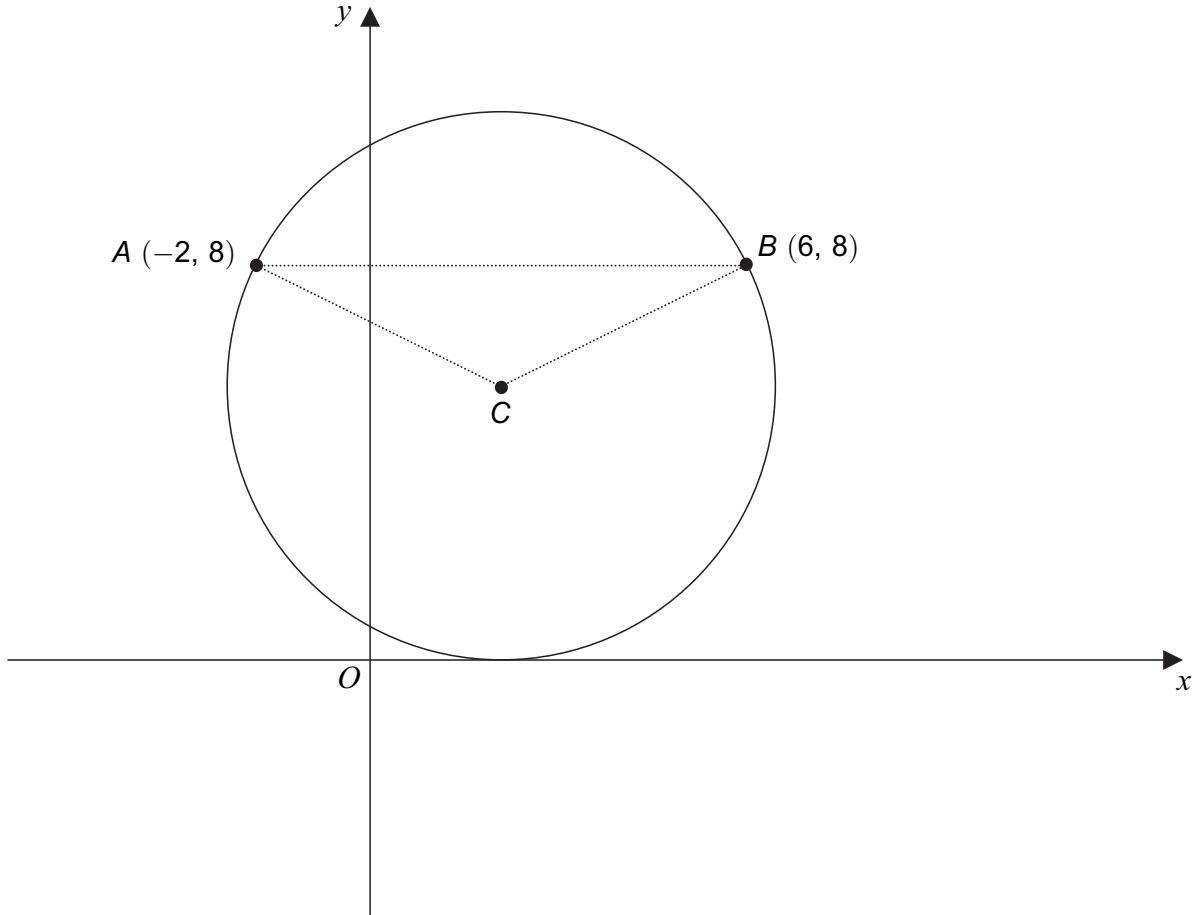
12
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Turn over ►



14

The sketch shows a circle, centre  $C$ , radius 5.  
The circle passes through the points  $A (-2, 8)$  and  $B (6, 8)$ .  
The  $x$ -axis is a tangent to the circle.



Work out the equation of the circle.

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Answer..... (4 marks)



**15 (a)**  $f(x) = 3x - 5$  for all values of  $x$ .

Solve  $f(x^2) = 43$

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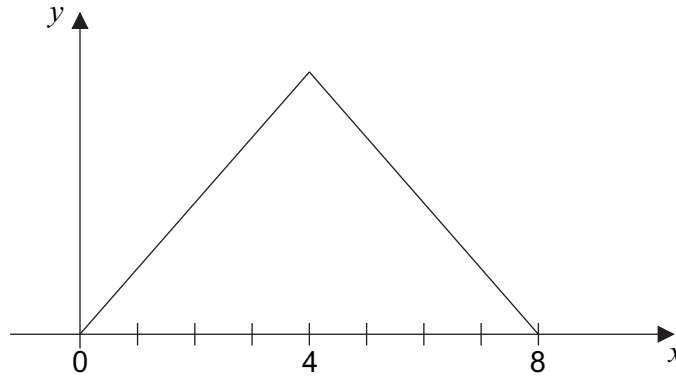
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Answer..... (4 marks)

**15 (b)** A sketch of  $y = g(x)$  for domain  $0 \leq x \leq 8$  is shown.



The graph is symmetrical about  $x = 4$   
The range of  $g(x)$  is  $0 \leq g(x) \leq 12$

Work out the function  $g(x)$ .

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$g(x) =$  .....  $0 \leq x \leq 4$   
.....  $4 < x \leq 8$

(5 marks)



**16 (a)** Use the factor theorem to show that  $(x - 1)$  and  $(x - 4)$  are factors of  $x^3 - 21x + 20$

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(2 marks)

**16 (b)** Show that  $(x - 1)$  and  $(x - 4)$  are also factors of  $x^3 - 10x^2 + 29x - 20$

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(2 marks)

**16 (c)** Hence, simplify fully  $\frac{x^3 - 21x + 20}{x^3 - 10x^2 + 29x - 20}$

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Answer..... (3 marks)

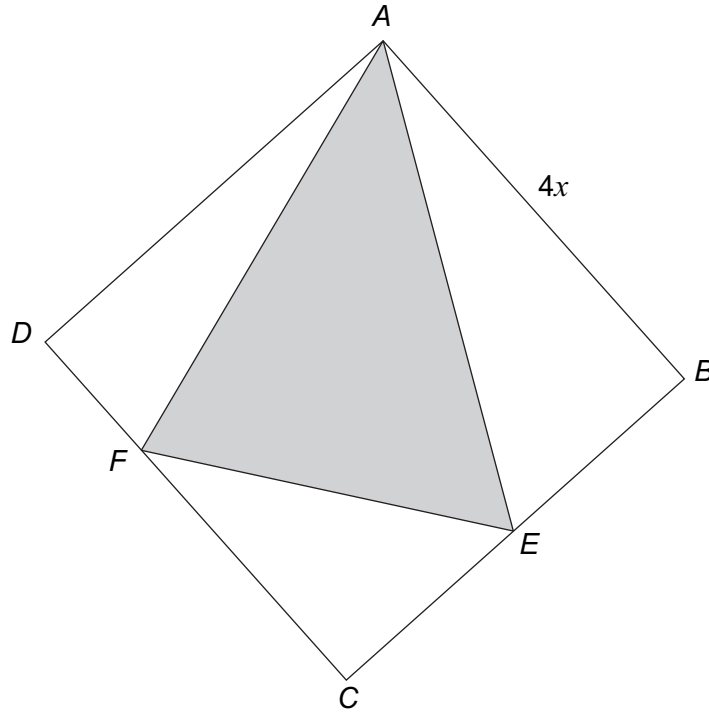


17

$ABCD$  is a square of side length  $4x$ .

$E$  is the midpoint of  $BC$ .

$DF:FC = 1:3$



Not drawn  
accurately

You are given that

$$\text{area of triangle } AEF = kx^2$$

Work out the value of  $k$ .

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$k = \dots\dots\dots$  (5 marks)

Turn over ►



18

$$(x - 5)^2 + a \equiv x^2 + bx + 28$$

Work out the values of  $a$  and  $b$ .

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$a = \dots\dots\dots b = \dots\dots\dots$  (3 marks)

19

Solve the simultaneous equations

$$x + y = 4$$

$$y^2 = 4x + 5$$

Do **not** use trial and improvement.

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Answer..... (6 marks)





**20** For what values of  $x$  is  $y = 150x - 2x^3$  an increasing function?

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Answer..... (4 marks)

**Turn over for the next question**



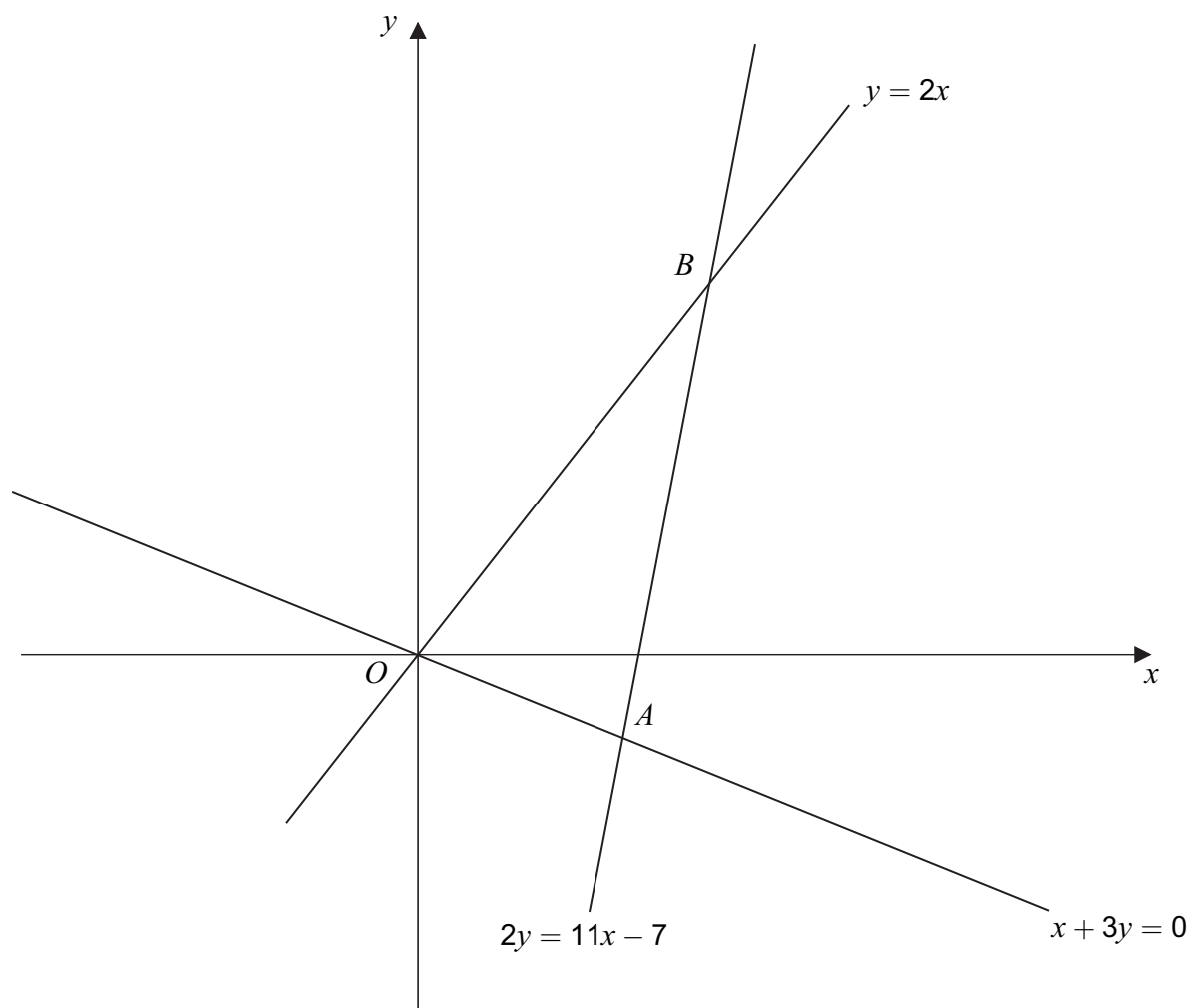
21 The equations of three straight lines are

$$y = 2x$$

$$x + 3y = 0$$

$$2y = 11x - 7$$

The lines intersect at the points  $O$ ,  $A$  and  $B$  as shown on this sketch.



Show that  $\text{length } OB = \text{length } AB$

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(6 marks)

**Turn over for the next question**



22 The transformation matrix  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$  maps point  $P$  to point  $Q$ .

The transformation matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  maps point  $Q$  to point  $R$ .

Point  $R$  is  $(-4, 3)$ .

Work out the coordinates of point  $P$ .

Answer ( ..... , ..... )

(5 marks)



23 The curve  $y = f(x)$  is such that  $\frac{dy}{dx} = -x(x - 2)^2$

The stationary points of the curve are at  $\left(0, \frac{4}{3}\right)$  and  $(2, 0)$ .

Determine the nature of each stationary point.  
You **must** show your working.

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(4 marks)

**END OF QUESTIONS**



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